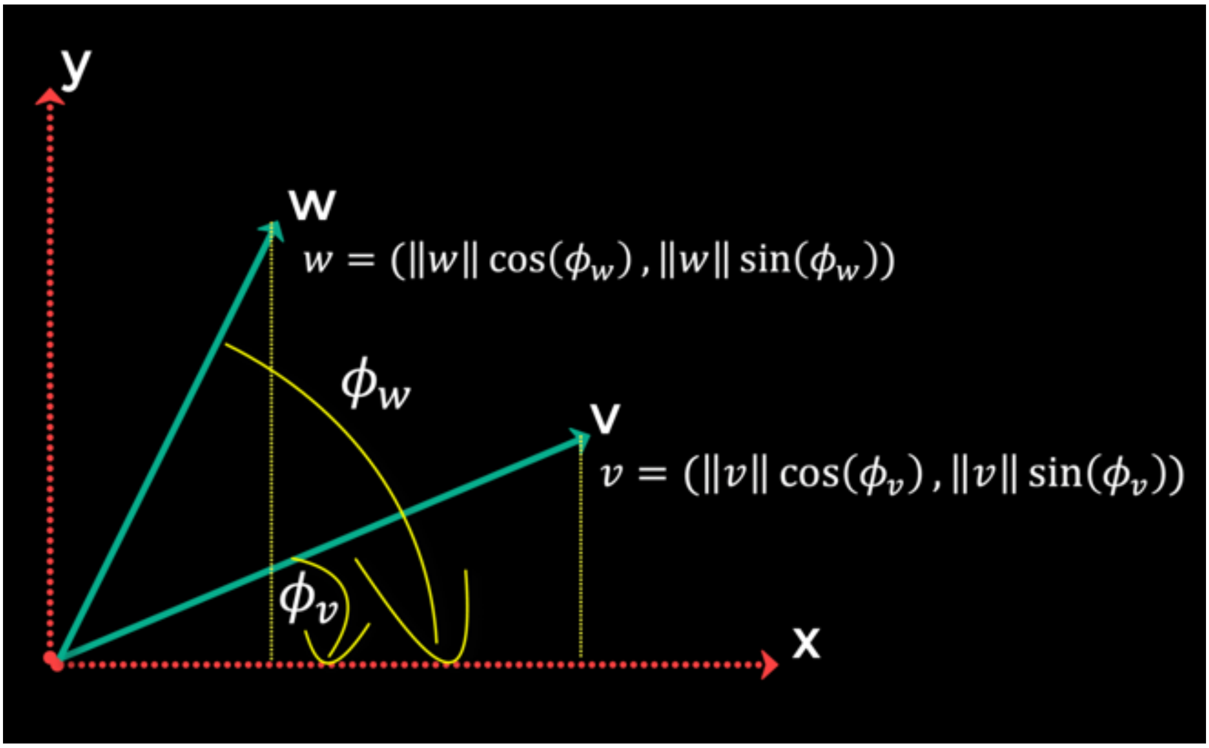
The Angle Between Two Vectors

Watch the lecture from this section until this derivation is referred to.



Given two vectors we can use the principles of a right-angle triangle to determine the x and y components where the x component is

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x = hypotenuse * cos (angle) &
y = hypotenuse * sin (angle)
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The hypotenuse is this case will be the length of the vector, x will be the length of the adjacent side and y the length of the opposite side. We can then rewrite the vector coordinates with

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x = length of vector * cos(angle)
y = length of vector * sin(angle)
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This allows us to rewrite the coordinates for *v* and *w* in the diagram above as:

$$v = (\|v\| \cos(\phi_v), \|v\| \sin(\phi_v))$$

$$w = (\|w\| \cos(\phi_w), \|w\| \sin(\phi_w))$$

That makes the dot product:

v.x * w.x   v.y * w.y

$$v.w = \boxed{\|v\| \cos(\phi_v) \times \|w\| \cos(\phi_w)} + \boxed{\|v\| \sin(\phi_v) \times \|w\| \sin(\phi_w)}$$

This can be reduced using further trigonometry to:

$$v.w = \|v\|\|w\| \cos(\phi_w - \phi_v)$$

and finally, to

$$v \cdot w = \|v\|\|w\| \cos(\theta)$$

or for better ease of coding the angle between two vectors can be found with:

$$\theta = cos^{-1}(\frac{v \cdot w}{\|v\|\|w\|})$$